

# The metals and insulators contained in massive gravity



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Based on [arXiv:1411.1003](https://arxiv.org/abs/1411.1003)  
with Matteo Baggioli

GReCO, IAP Paris  
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# Motivation



## 1) Massive Gravity

Many **consistent** phases of MG in the market.

- **dRGT – Lorentz Invariant (??)**

- **Rubakov-Dubovsky 2004 – Lorentz-breaking**

Relevance for cosmology (3+1 gravity) **dubious-unclear**

Relevant for ‘anti-cosmology’ i.e. AdS/CFT duality  
without Lorentz-inv → Cond Matt

# Motivation



## 2) Condensed Matter

Finite density ( $\rho$ ) and strong interactions  $\Rightarrow$  **tough problem**

$\exists$  plethora of puzzling materials. Often layered (2+1)

HTSC {  
Strange metals (Non Fermi-liquids),  
Interaction-driven **insulators** (Mott, Anderson localization)

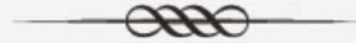
**Strong dynamics crucial**  $\rightarrow$  AdS/CMT

# Plan



- ∞ **Motivation**
- ∞ **Cond Matt crash course**
- ∞ **AdS/CFT crash course**
- ∞ **Massive Gravity renders AdS/CMT realistic**
- ∞ **Metal-Insulator transitions**

# CM crash course

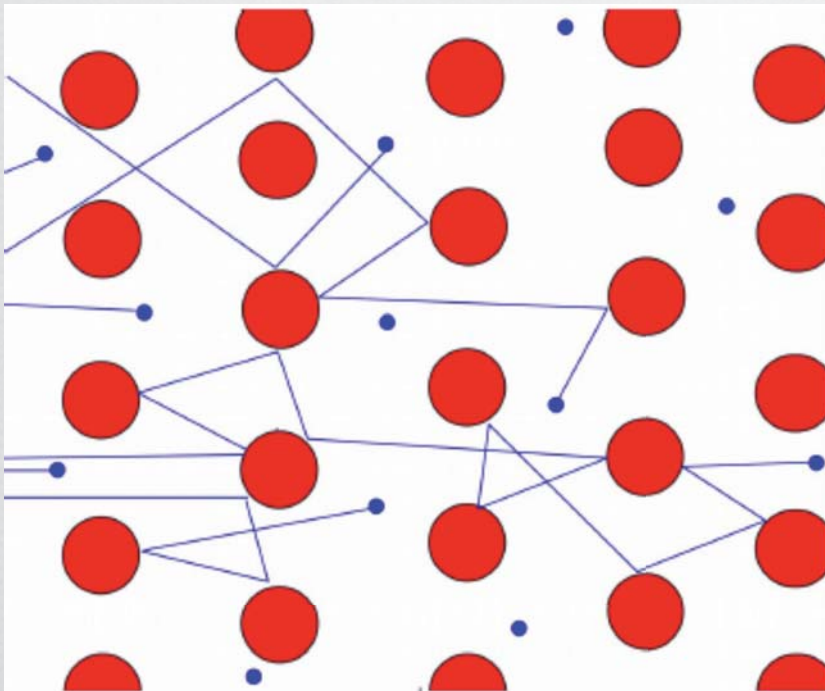


**Electrical conductivity (metals, insulators, ...)**

# CM crash course



## Drude model



$$m_* \left[ \dot{\vec{v}} + \frac{\vec{v}}{\tau} \right] = e \vec{E}$$

$$\Rightarrow \left( \vec{J} = \rho \vec{v} \right)$$

$$\sigma_{DC} = \frac{e \tau}{m_*} \rho$$

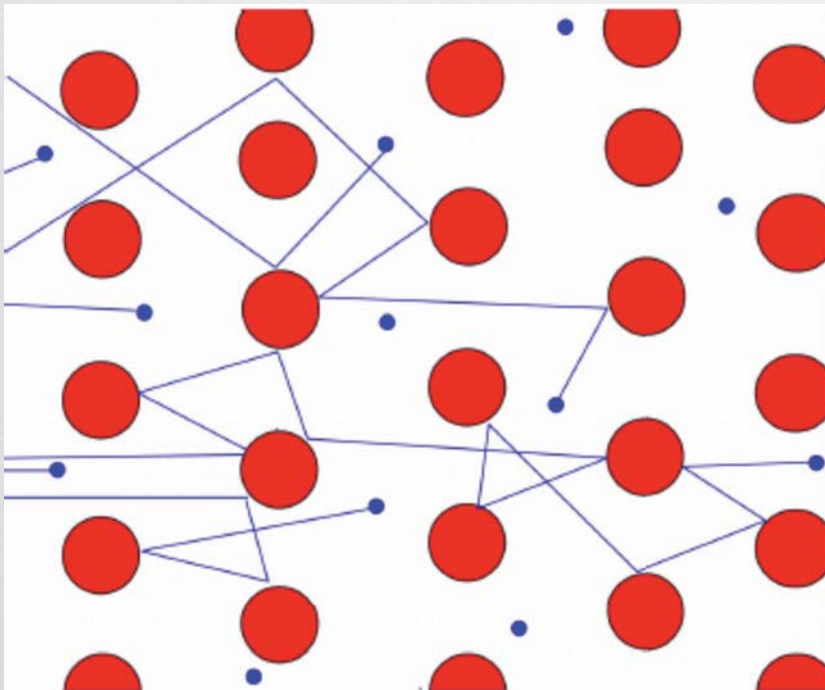
$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}$$

# CM crash course



$\tau$  : collision time  
momentum non-conservation

## Drude model



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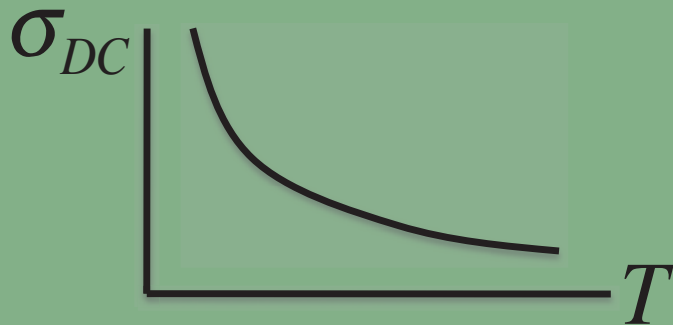
# CM crash course



$$\sigma_{DC} = \frac{e\tau}{m_*} \rho$$

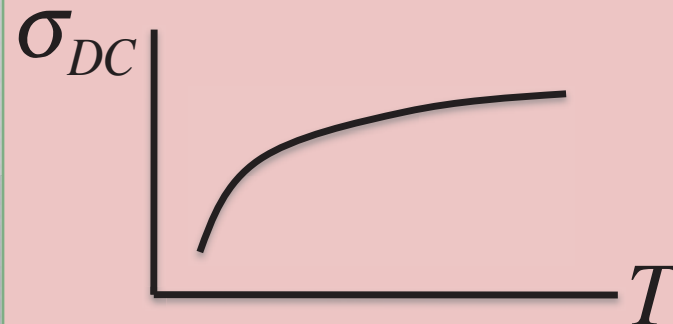
Metal:  $\sigma$  large

$$\frac{d\sigma}{dT} < 0$$



Insulator:  $\sigma$  small

$$\frac{d\sigma}{dT} > 0$$





# CM crash course

Question marks:  $(\sigma_{DC} = \frac{e\tau}{m_*} \rho)$

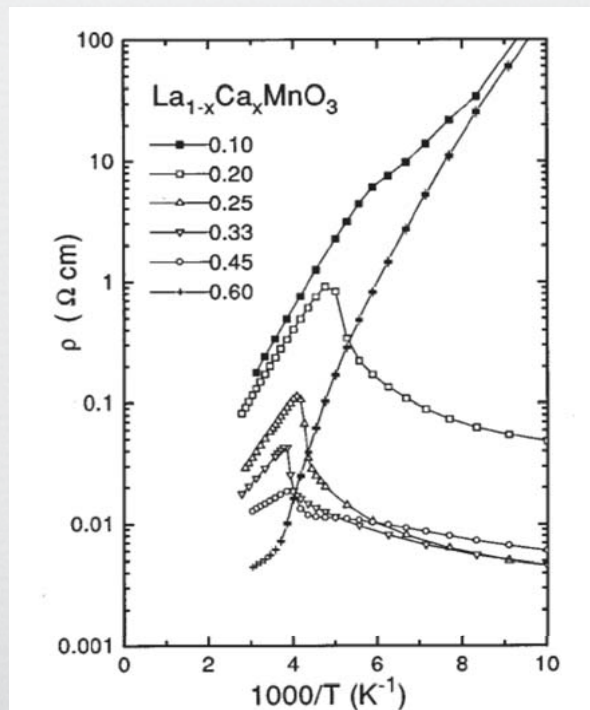
- *Correlated insulator*:  $\rho$  large but  $\sigma$  small
- Materials with **Metal-Insulator transition**

some HTSCs

cuprates,

manganites,

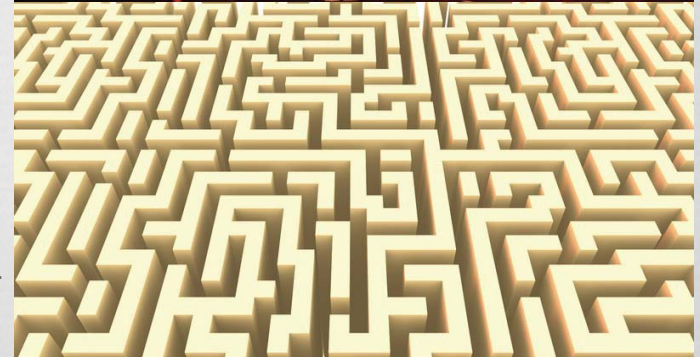
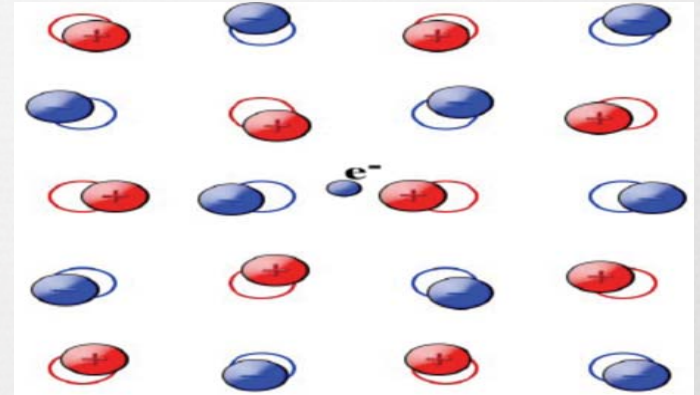
perovskites, ...



# CM crash course

Various mechanisms:

- i) Polaron (electron-phonon int.)
- ii) Mott-Wigner (electron-electron)
- iii) Disorder / Anderson localization



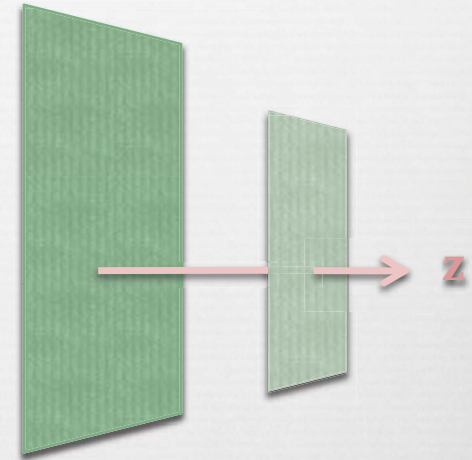
# AdS/CFT crash course



# AdS/CFT crash course

Physics  
(boundary  
conditions)  
in AdS ??

$$ds_{(d+1)}^2 = \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$



$$\Phi(z, x) \simeq \Phi_-(x) z^{\Delta_-} + \Phi_+(x) z^{\Delta_+} + \dots$$

$$J(x)$$

$$\hat{O}(x)$$

$$\Delta_+ + \Delta_- = d$$

$$S_{on-shell} = \int d^d x \sqrt{h} (\dots + \Phi_- \Phi_+ + \dots) = \log[Z(J)]$$

**QFT interpretation in terms of boundary data  
for a strongly coupled CFT**

$$z \sim \frac{1}{\mu}$$

# AdS/CFT crash course

- *QFTs with gravity dual* → **dynamics simplify enormously**

few QFT operators =  $\{ T_{\mu\nu}, J_{\mu}, O, \dots \}$

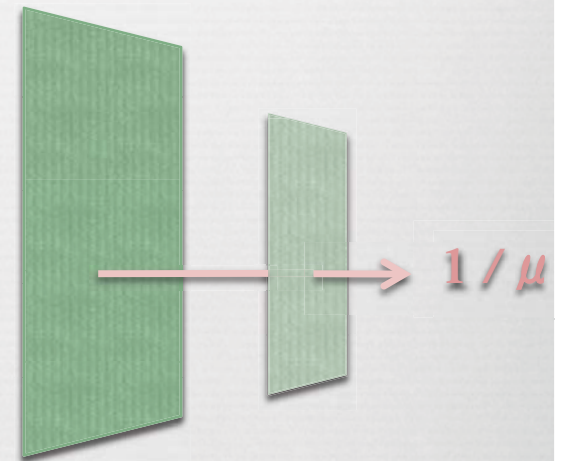
→ Finite **T**,  **$\rho$**  problems easily tractable

- Self-consistent dynamics:

$$T_{\mu\nu}^{CFT} \subset g_{\mu\nu}$$

$$J_{\mu}^{CFT} \subset A_{\mu} \quad \text{with nonlinearities}$$

- spacetime symmetries → holographic isometries
- Many non-trivial QFT effects: **nonperturbative RG flows**, **unparticle behaviour (broad resonances)**, **collective effects**, **emergent symmetries & DOFs**, **dissipation** in QFT, **SSB**, ...



# **Realistic AdS/ CMT ?**

# Unrealistic AdS/CMT

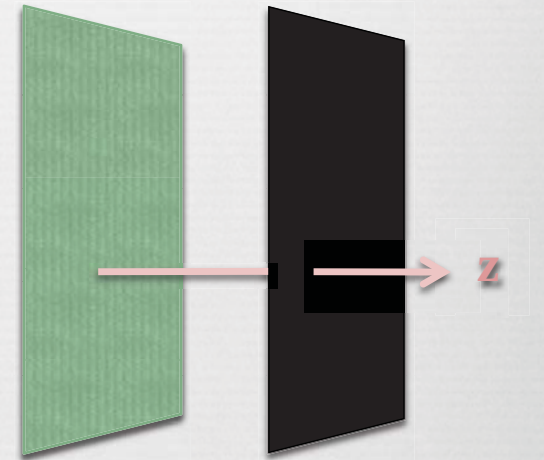
CFT at Finite  $T$ ,  $\rho$ : RN AdS BB

$$ds^2 = \frac{\ell^2}{z^2} \left[ \frac{dz^2}{f(z)} - f(z) dt^2 + dx_{d-1}^2 \right]$$

$$f(z) = 1 - \varepsilon z^3 + \rho^2 z^4$$

$$A_t(z) = \mu - \rho z$$

charge-density (density of mobile charge-carriers)



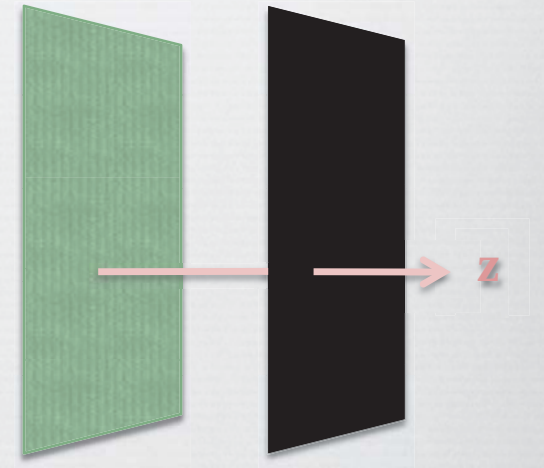
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Conductivity:  $A_i = A_i^{ext}(t) + z J_i + \dots$

$$\sigma(\omega) = \frac{J}{i\omega A^{ext}} \Rightarrow \sigma(0) = \infty!!$$



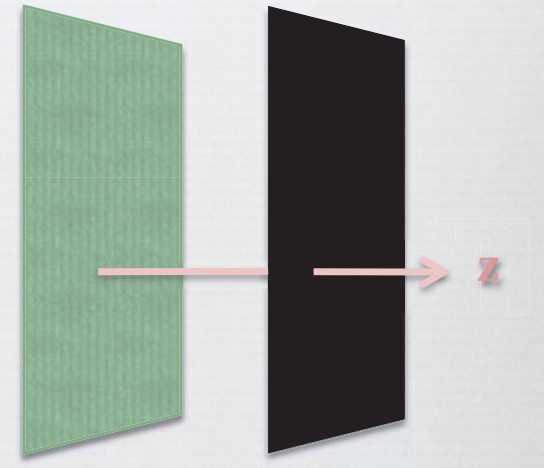
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need to relax  $\vec{P}$   
 $\rightarrow$  break translations

# Realistic AdS/CMT

$$S = \int \sqrt{-g} \left\{ R - 2\Lambda - F_{\mu\nu}^2 + m^2 V(g^{\mu\nu}) \right\}$$

graviton mass => **new degrees of freedom**  
**PHONONS**

# Realistic AdS/CMT

$$S = \int \sqrt{-g} \left\{ R - 2\Lambda - F_{\mu\nu}^2 + m^2 V(X) \right\}$$

Phonon-dynamics

$$X \equiv \partial_\mu \Phi^I \partial_\nu \Phi^I g^{\mu\nu}$$

$$\langle \Phi^I \rangle = \delta_i^I x^i$$

# Realistic AdS/CMT

$$S = \int \sqrt{-g} \left\{ R - 2\Lambda - F_{\mu\nu}^2 + m^2 V(X) \right\}$$

## Phonon-dynamics

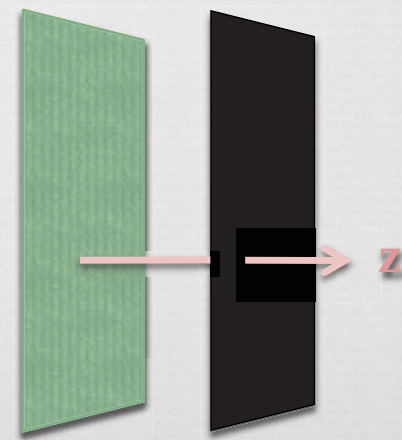
There are BB solutions with  
'hedgehog' scalar charge

$$ds^2 = \frac{\ell^2}{z^2} \left[ \frac{dz^2}{f(z)} - f(z) dt^2 + dx_{d-1}^2 \right]$$

$$f(z) = 1 - \varepsilon z^3 + \rho^2 z^4 + \dots$$

$$X \equiv \partial_\mu \Phi^I \partial_\nu \Phi^I g^{\mu\nu}$$

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# Realistic AdS/CMT

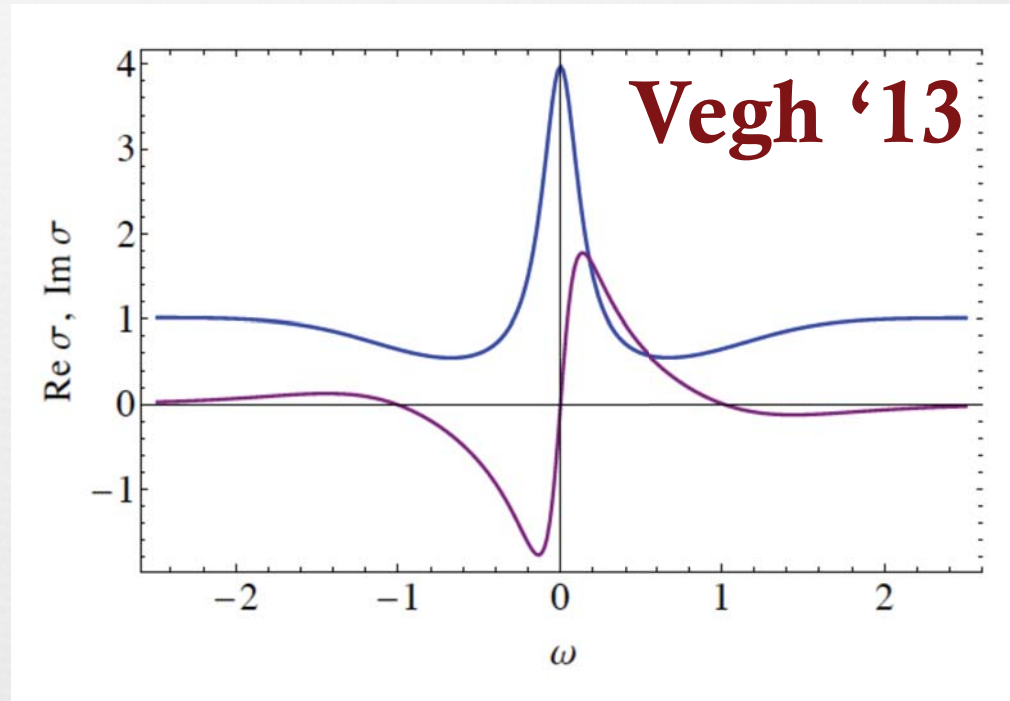
Conductivity:

$$\sigma(\omega) \simeq \frac{\sigma_{DC}}{1 - i\omega\tau}$$

$$\sigma_{DC} = 1 + \frac{\rho^2 z_H^2}{m^2}$$

Just with a  
graviton mass term,  $V(X)=X$

$$\tau \sim \frac{1}{m^2}$$



# Realistic AdS/CMT

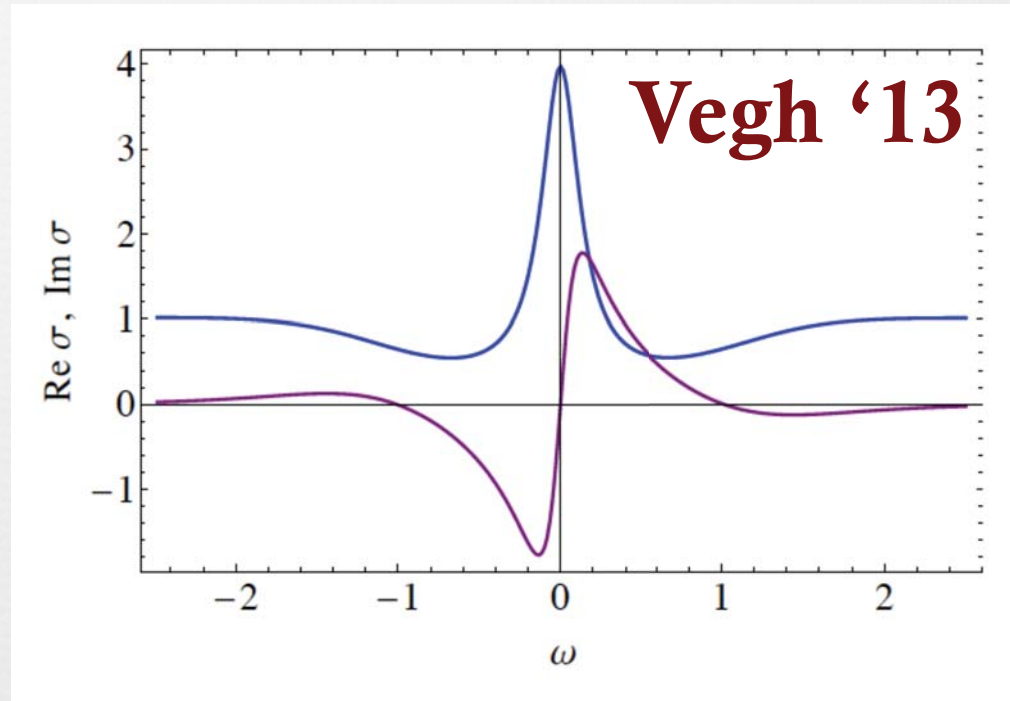
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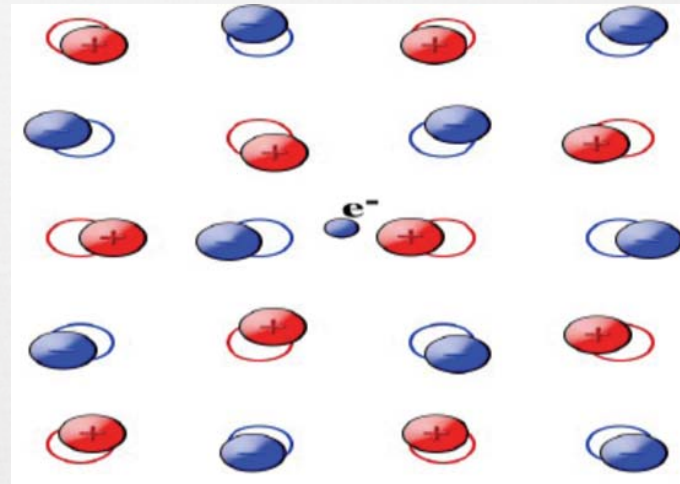
Just with a  
graviton mass term

$$\tau \sim \frac{1}{m^2}$$



# Metal-Insulator transitions

i) Polaronic



ii) Mott-Wigner



# Metal-Insulator transitions

$$\text{i) } S = \int \sqrt{-g} \left\{ R - 2\Lambda - F_{\mu\nu}^2 + m^2 V(X) \right\}$$

Quite general form  
of  $V(X)$  allowed

$$V'(X) > 0$$

$$XV''(X) + V'(X) > 0$$

$$\Rightarrow \left\{ \begin{array}{l} \Phi^I = \langle \Phi^I \rangle + \delta\Phi^I \quad M_{\delta\Phi}^2 \approx -\frac{V''(z_H^2)}{V'(z_H^2)} < 0 \quad \text{Polaron formation} \\ \sigma_{DC} = 1 + \frac{\rho^2 z_H^2}{m^2 V'(z_H^2)} \quad \text{Metal-Insulator transition} \end{array} \right.$$



# Metal-Insulator transitions

Debye / plasma mass term

$$\partial_u (f \partial_u a_i) + \left[ \frac{\omega^2}{f} - k^2 - 2u^2 \rho^2 \right] a_i = \frac{i \rho u^2 (2\bar{m}^2 + k^2)}{\omega} U_i - \frac{i f \rho k^2}{\omega} \partial_u B_i,$$

$$\frac{1}{u^2} \partial_u \left[ \frac{f u^2}{\bar{m}^2} \partial_u (\bar{m}^2 U_i) \right] + \left[ \frac{\omega^2}{f} - k^2 - 2\bar{m}^2 \right] U_i = -2i \rho \omega a_i + \frac{f' k^2}{u^2} B_i,$$

$$k \left\{ u^2 \partial_u \left( \frac{f}{u^2} \partial_u B_i \right) + \left[ \frac{\omega^2}{f} - k^2 - 2\bar{m}^2 \right] B_i = -2 \frac{\bar{m}'}{\bar{m}} U_i \right\},$$

bulk linearized equations for vector modes

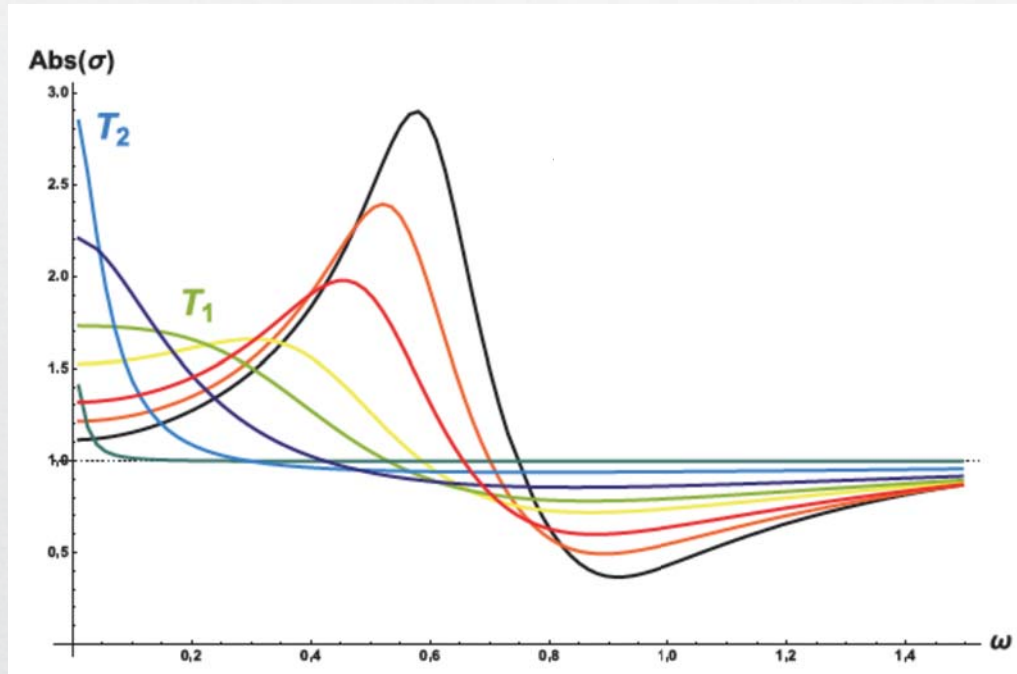
$$T_i \equiv u^2 h_{ti} - \frac{\partial_t \phi_i}{\alpha}, \quad U_i \equiv f(u) \left[ h_{ui} - \frac{\partial_u \phi_i}{\alpha u^2} \right], \quad B_i \equiv b_i - \frac{\phi_i}{\alpha}$$

gauge invariant variables

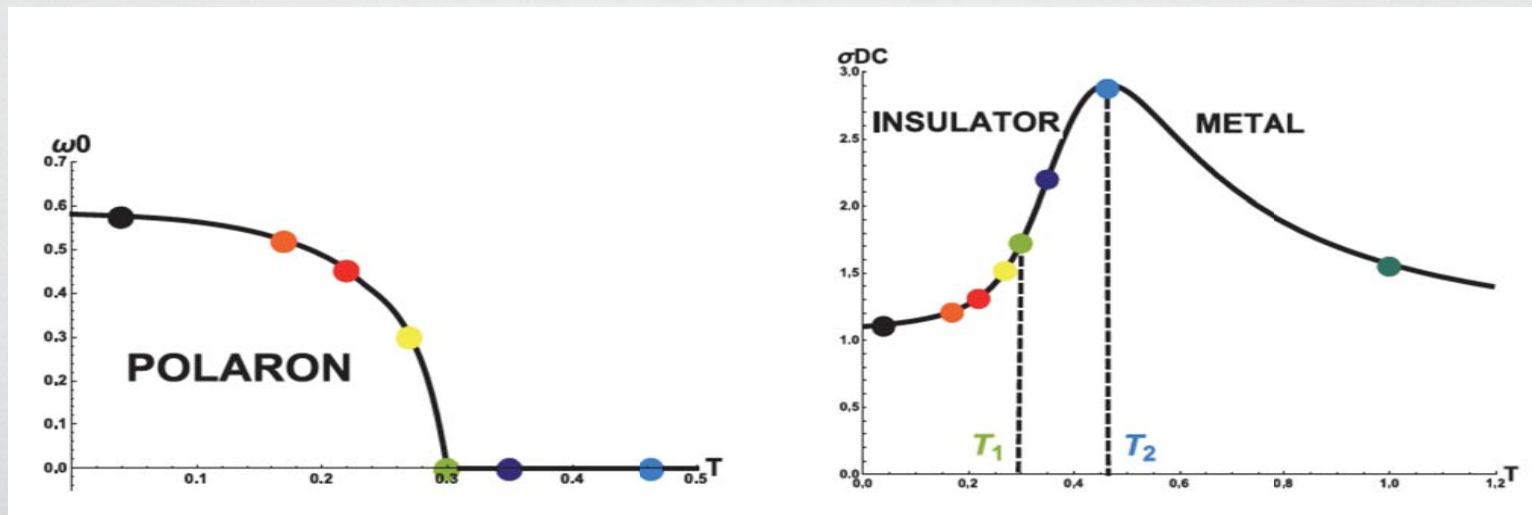
$$\bar{m}^2(u) = \alpha^2 m^2 V'(\alpha^2 u^2)$$

# Metal-Insulator transitions

i)

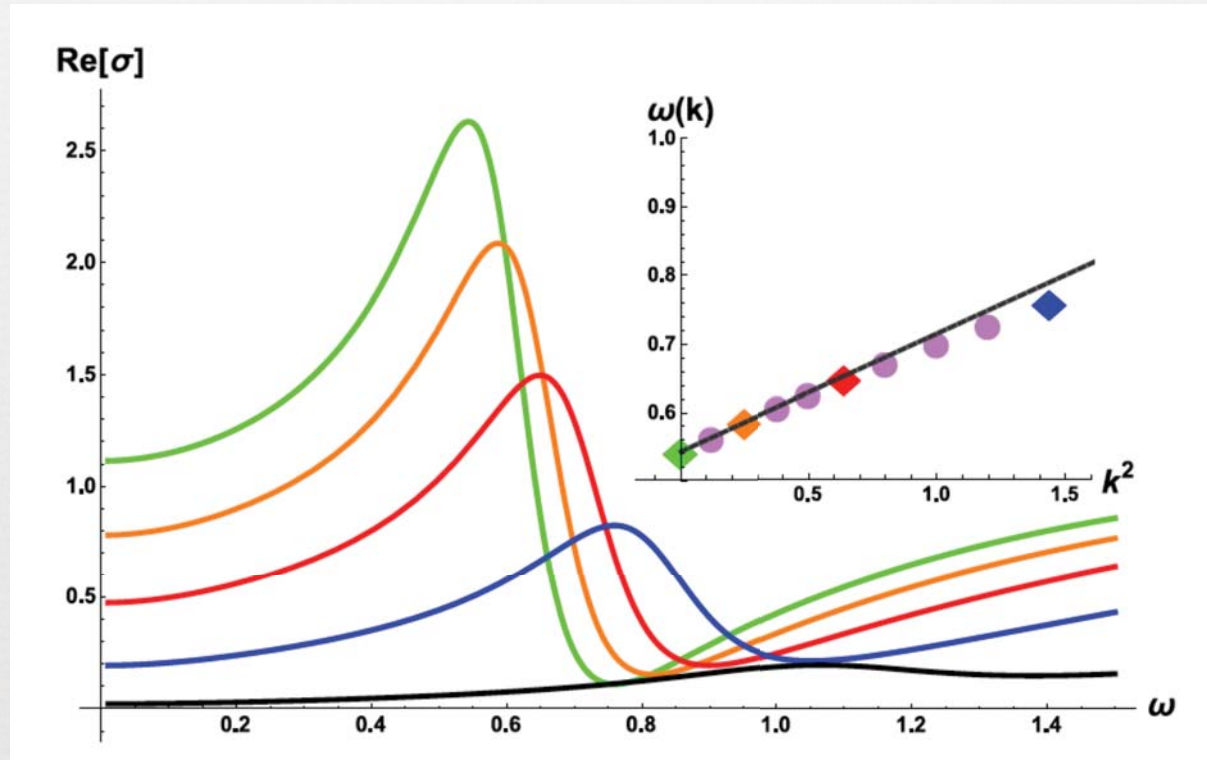


electrons  
trapped by large  
lattice  
deformations  
 $\rightarrow m_*$  huge



# Metal-Insulator transitions

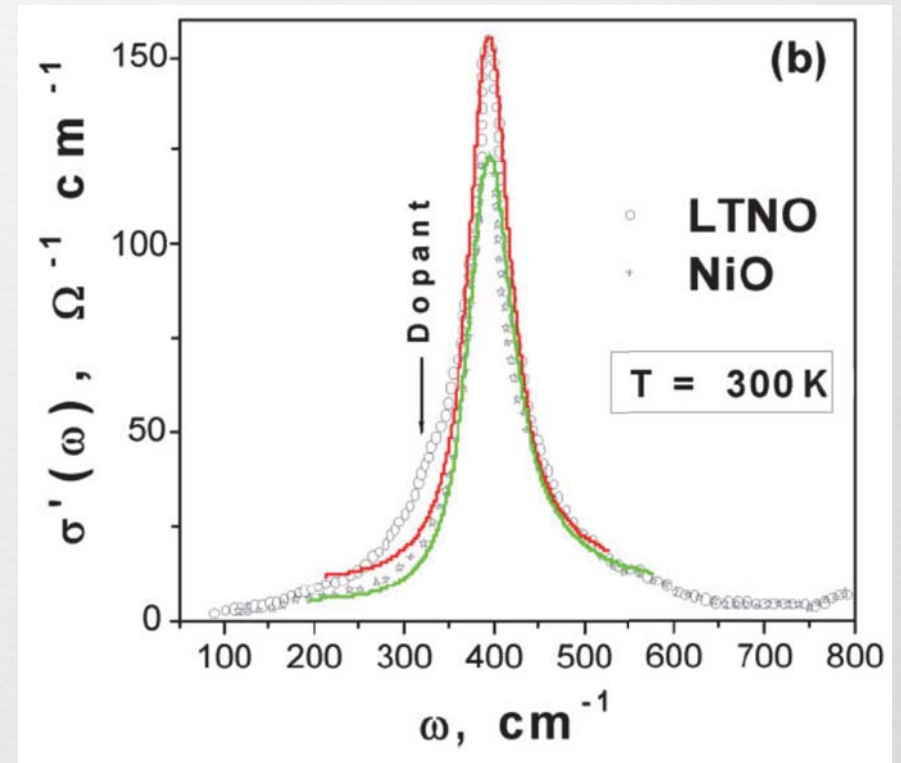
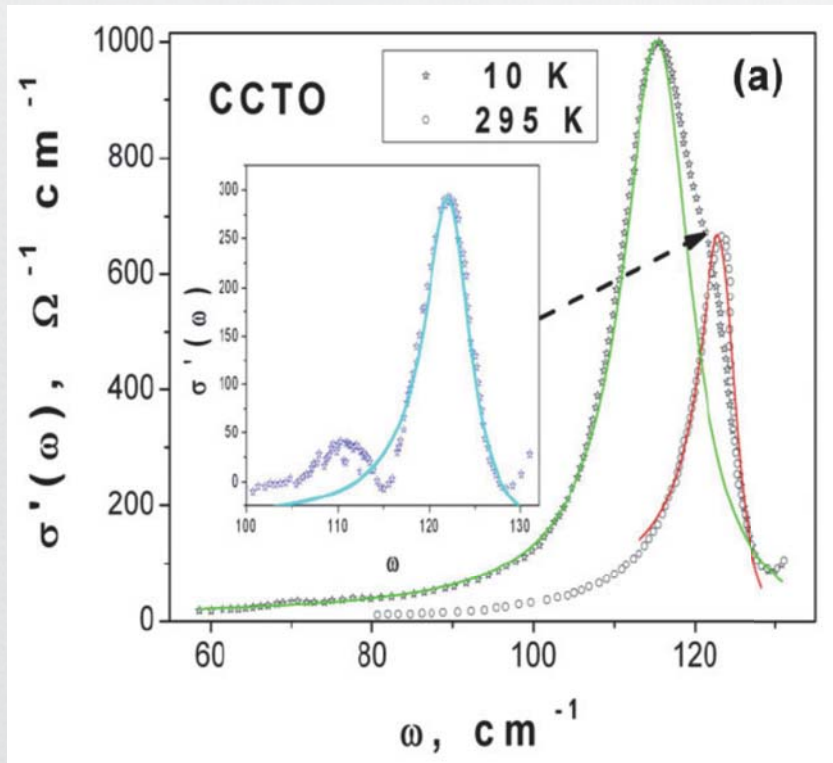
i)



the **polaron** is an **emergent** particle degree of freedom with a well defined dispersion relation, mass, width, ... (we started from a CFT)

# Polarons in the real world

i)



Phys. Status Solidi B 251, No. 3, 569–592 (2014)  
Valeri Ligatchev



# Polarons in the real world

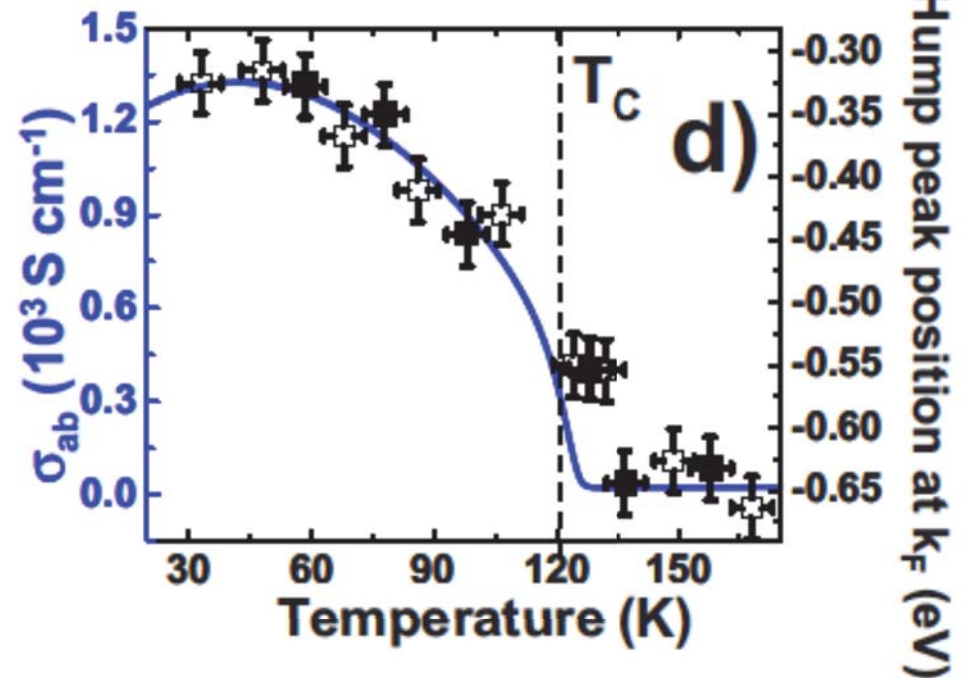
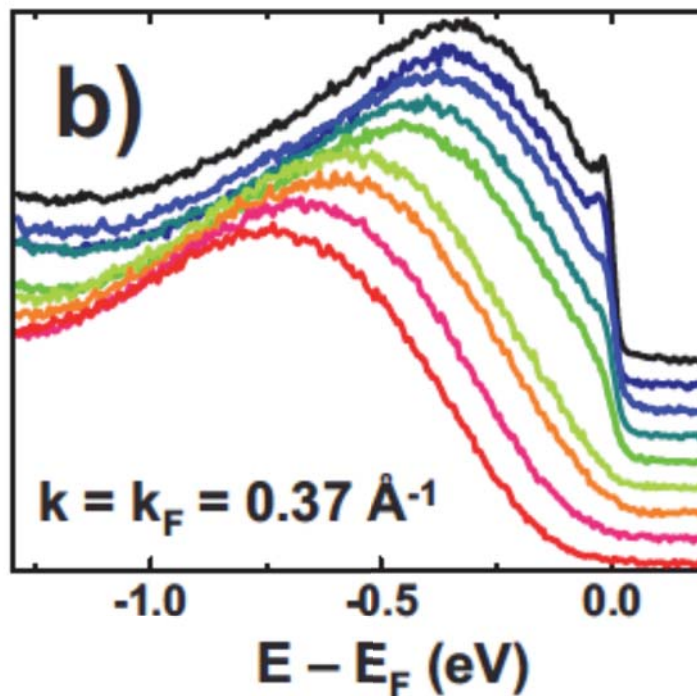
i)

**Polaron coherence condensation as the mechanism for colossal magnetoresistance in layered manganites**

N. Mannella, PHYSICAL REVIEW B 76, 233102 (2007)

T (K) =

- 33
- 48
- 68
- 86
- 106
- 124
- 132
- 149
- 168



# Conclusions

- ∞ **Massive Gravity** has a real-world application !
- ∞ Charged MG AdS BHs know about **CM** !
- ∞ Many consistent phases of LV MG out there
- ∞ Phases of MG -> phases of Holography?
- ∞ AdS/CMT **predictability**
  - identify robust correlations (in progress)

**Thanks!**